MATH 147 Review: Taylor Series

1. Find the Taylor Series of the function $f(x) = x^2 - 2x + 1$ centered at 1.

Solution:

$$x^2 - 2x + 1 = (x-1)^2$$

2. Find the Taylor Series of the function $f(x) = \frac{1}{1+x^2}$ centered at 0.
Hint: $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$.

Solution:

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n}$$

3. Find the Taylor Series of the function $f(x) = \tan^{-1}(x)$ centered at 0.
Hint: $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$.

Solution:

$$\tan^{-1}(x) = \int \frac{1}{1+x^2}\,dx = \int \sum_{n=0}^{\infty}(-1)^n x^{2n}\,dx = \sum_{n=0}^{\infty}\int (-1)^n x^{2n}\,dx = \sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}x^{2n+1}$$